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# Parametric studies of the dynamic stability of submersible

Cunningham, Stanley

Monterey, California. Naval Postgraduate School

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PARAMETRIC STUDIES OF THE DYNAMIC STABILITY OF SUBMERSIBLES

by

Stanley Cunningham

March, 1994

Thesis Advisor: Fotis A. Papoulias

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# PARAMETRIC STUDIES OF THE DYNAMIC STABILITY OF SUBMERSIBLES

by


Stanley Cunningham
Lieutenant Commander, United States Navy
B.S., South Carolina State College, 1982


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March 1994

Author: [redacted]
Stanley Cunningham

Approved by: [redacted]
Fotis A. Papoulias, Thesis Advisor

[redacted]
Matthew D. Kelleher, Chairman
Department of Mechanical Engineering

## ABSTRACT

This thesis analyzes the dynamic stability of submersible vehicles in motions in six degrees of freedom. A continuation algorithm is used in order to obtain the steady state solutions in terms of dive plane angle, rudder angle, and longitudinal separation of centers of gravity/buoyancy. The equations of motion are then linearized in the vicinity of the above stated nominal point. The eigenvalues of the linearized system indicate the degree of stability of the nominal motion. The results demonstrate the stabilizing or destabilizing effects of general three dimensional motions as opposed to the traditional use of straight line level flight paths. Recommendations for robust design of control laws for commanded paths in combined horizontal/vertical planes are provided.

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# I. INTRODUCTION

## A. GENERAL

As the versatility of underwater unmanned submersibles are becoming imagined and realized, the question of dynamic stability must be addressed and justified. Salvage, survey, and rescue operations as well as defense related operations of the future may become increasingly dependent on the ability of reliable unmanned underwater submersibles to perform prescribed missions. Theoretically, an unmanned submersible vehicle would not be as physically restricted in its motion as a manned vehicle. Because of its inhabitants and cargo, a manned submersible is limited mostly to straight line motion. This thesis seeks to explore the dynamic stability of submersible vehicles in motions in six degrees of freedom.

The tight maneuvers that can be performed by an unmanned submersible vehicle make analysis of dynamic stability very difficult. The nonlinearity of the dynamic properties is very complicated and sometimes very hard to visualize. The most effective analysis of the dynamics and stability of a waterborne vehicle comes from observing and/or modelling the hydrodynamic forces acting upon it through the use of the translational and rotational equations of motion in six degrees of freedom. These equations are linearized in the

vicinity of a nominal flight path. The eigenvalues of the linearized set of equations are an indicator of the relative degree of dynamic stability for the vehicle's motion.

Given a mathematical model with an established set of physical characteristics, analysis of the dynamic stability can be studied varying the dive plane angle, the rudder angle, and the location of the center of gravity. Analysis of the linearized equations of motion and determination of the eigenvalues by individual calculation would prove tedious. Therefore, computer algorithms developed by Seydel [Ref.1] and Aydin [Ref.2] are used in this thesis to accomplish these tasks. An explanation of uncoupled stability is provided in Chapter II. Formulation of the linearized equations of motion in six degrees of freedom is presented in Chapter III. Chapters II and III are based and partially paraphrased from Papoulias [Ref.3]. The experimental results in Chapter IV are based on the understanding of Chapters II and III. Chapter V summarizes the results and provides recommendations for future submersible modelling research.

## B. PARAMETERS

Several variables were used in the algorithms associated with this research. The shorthand notation is from Smith, Crane and Summey [Ref.4] and is as follows:

**Variables**

x, y, z Distances along the body fixed axes.

| | |
|---|---|
| u, v, w | Translational velocity components of model relative to fluid along body axes. |
| p, q, r | Rotational velocity components of model along body axes. |
| X, Y, Z | Hydrodynamic force components along body axes. |
| K, M, N | Hydrodynamic moment components along body axes. |
| $\psi$, $\theta$, $\phi$ | Yaw, pitch, and roll angles; positive values following the right hand rule. |
| $x_G$, $y_G$, $z_G$ | Center of gravity coordinates along body axes. |
| $x_B$, $y_B$, $z_B$ | Center of buoyancy coordinates along body axes. |
| $I_{xx}$, $I_{yy}$, $I_{zz}$ | Moments of inertia about body axes. |

The standard body-fixed, right-hand orthogonal axis system is employed throughout all data simulations. All data run simulations described herein were initialized with a constant propeller speed of 500 rpm, which translates to a value of 6.0 ft/sec for "u". The data is analyzed for rudder angles between 0 and 20 degrees and stern diving plane angles between -20 and 20 degrees. Separate data runs were conducted within the above stated parameters for simulated model configurations with the center of gravity shifted longitudinally forward and aft between -1.5 and 1.5 percent of the model length.

## II. UNCOUPLED STABILITY

### A. MANEUVERING IN THE HORIZONTAL PLANE

#### 1. Basic Maneuvering

It is extremely important to note that the ability to control a vessel involves two major characteristics that are contrary to one another. These two characteristics are the ability of controls fixed course stability and the ability of the vessel to turn. Obviously a vessel that is extremely course stable will not turn easily. Newton's equations of motion, used with reference frames both fixed with respect to the earth (inertial) and fixed relative to the vessel will illustrate the dynamics of ship motion. This motion will be first described in the horizontal plane and later described in the vertical plane. As a body moves within the inertial frame, its motions can be described as follows:

$$m\,\ddot{x}_o = X_o \tag{1}$$

$$m\,\ddot{y}_o = Y_o \tag{2}$$

$$I_z\,\ddot{\psi} = N \tag{3}$$

through the use of the variables introduced in Chapter I. The subscript o pertains to motion in the inertial reference

frame. In the vessel's relative frame of reference, these equations become:

$$X = X_o \cos\psi + Y_o \sin\psi \,, \tag{4}$$

$$Y = Y_o \cos\psi - X_o \sin\psi \,, \tag{5}$$

$$\dot{x}_o = u \cos\psi - v \sin\psi \,, \tag{6}$$

$$\dot{y}_o = u \sin\psi + v \cos\psi \,, \tag{7}$$

and

$$\dot{\psi} = r \,. \tag{8}$$

Therefore, equations (6) and (7) can be integrated and substituted into equations (1) through (3). They in turn can be substituted into equations (4) and (5) with the following results:

$$m\dot{u} - mvr = X \,, \quad \textit{surge equation,} \tag{9}$$

$$m\dot{v} + mur = Y \,, \quad \textit{sway equation,} \tag{10}$$

$$I_z \dot{r} = N \,, \quad \textit{yaw equation.} \tag{11}$$

These equations are true only if the reference plane center and the vessels center of gravity coincide. If the two are indeed located at different locations, the following equations result:

$$m\dot{u} - mvr - mx_G r^2 = X , \tag{12}$$

$$m\dot{v} + mur + mx_G \dot{r} = Y , \tag{13}$$

$$I_z \dot{r} + mx_G (\dot{v} + ur) = N , \tag{14}$$

provided that $y_G = 0$. An important point to note is that centrifugal forces do not appear in equations (1) through (3). These forces are not evident until the relative reference plane is used in equations (9) through (12). Centrifugal forces are not experienced on the inertial reference frame since it is stationary but are indeed evident when the relative reference plane of the vessel is studied. The hydrodynamic forces X and Y and the moment $N$ are composed of four different types of forces:

1. F forces; Fluid forces acting on the hull by the surrounding fluid (water).

2. R forces; Control surface forces due to rudders, bow planes, dive planes, thruster, etc.

3. E forces; Environmental forces due to wind, current and waves.
4. T forces; Propulsion forces due to propellers, thrusters etc.

Due to the construction characteristics of most waterborne vessels today, the following can be stated:

$$X = X_F + X_R + X_E + T \ , \tag{15}$$

$$Y = Y_F + Y_R + Y_E \ , \tag{16}$$

$$N = N_F + N_R + N_E \ . \tag{17}$$

Since the hydrodynamic forces $X_F$, $Y_F$, and $N_F$ depend on vessel motion and must therefore be functions of vessel velocity and acceleration through the water we can write:

$$X_F = X_F(u, v, \dot{u}, \dot{v}, r, \dot{r}) \ , \tag{18}$$

$$Y_F = Y_F(u, v, \dot{u}, \dot{v}, r, \dot{r}) \ , \tag{19}$$

$$N_F = N_F\ (u, v, \dot{u}, \dot{v}, r, \dot{r}) \ . \tag{20}$$

The interrelationship of equations (18) through (20) is a complicated one. Usual maneuvering studies simplify this interrelationship. The object is to study ship response about a nominal equilibrium point (designated by the subscript 1). Through the use of the Taylor series expansion around the nominal point, and keeping only the first order terms, the Y relationship becomes:

$$Y_F = Y_F(u_1, v_1, \dot{u}_1, \dot{v}_1, r_1, \dot{r}_1) + (u - u_1)\frac{\partial Y_F}{\partial u} + (v - v_1)\frac{\partial Y_F}{\partial v}$$

$$+ \ldots, (\dot{r} - \dot{r}_1)\frac{\partial Y_F}{\partial Y_{\dot{r}}} \quad , \tag{21}$$

where all of the partial derivatives are evaluated at a nominal set of conditions. $X_F$ and $N_F$ can be similarly manipulated and evaluated.

The nominal set of conditions is the set of variables associated with straight line motion:

$$u_1 = U, \; v_1 = \dot{u}_1 = \dot{v}_1 = r_1 = \dot{r}_1 = 0 \tag{22}$$

For the sake of understanding, the vessel is considered to be port/starboard symmetric. Therefore $\partial F/\partial u = \partial F/\partial \dot{u} = 0$ since

a change in the forward velocity or acceleration will produce no transverse force in vessels that are also symmetric in xz plane. In addition, if the vessel is indeed in equilibrium, and in straight line motion, there can be no $Y_F$-forces. Therefore the first term in equation (21) is also zero. Once these additional considerations are made, equation (21) reduces to:

$$Y_F = \frac{\partial Y_F}{\partial v} v + \frac{\partial Y_F}{\partial \dot{v}} \dot{v} + \frac{\partial Y_F}{\partial r} r + \frac{\partial Y_F}{\partial \dot{r}} \dot{r} \quad . \tag{23}$$

In like fashion, the surge force and the yaw moment can be simplified:

$$X_F = \frac{\partial X_F}{\partial \dot{u}} \dot{u} + \frac{\partial X_F}{\partial u} (u - U) + \frac{\partial X_F}{\partial v} v + \frac{\partial X_F}{\partial \dot{v}} \dot{v} + \frac{\partial X_F}{\partial r} r + \frac{\partial X_F}{\partial \dot{r}} \dot{r} , \tag{24}$$

$$N_F = \frac{\partial N_F}{\partial v} v + \frac{\partial N_F}{\partial \dot{v}} \dot{v} + \frac{\partial N_F}{\partial r} r + \frac{\partial N_F}{\partial \dot{r}} \dot{r} \quad . \tag{25}$$

Where the cross coupling derivatives $\partial Y_F/\partial r$, $\partial Y_F/\partial \dot{r}$, $\partial N_F/\partial v$, and $\partial N_F/\partial \dot{v}$ usually have small nonzero values because most vessels are not symmetrical about the yz plane even if that plane is at the midlength of the vessel (bow and stern contours are usually drastically different). However the

cross coupling derivatives $\partial X_u/\partial r$, $\partial X_F/\partial \dot r$, $\partial X_F/\partial v$, and $\partial X_F/\partial \dot v$ are zero because of the symmetry about the xz plane and equation (22). Equation (24) now reduces to:

$$X_F = \frac{\partial X_F}{\dot u}\dot u + \frac{\partial X_F}{u}(u - U) \quad . \tag{26}$$

Using standard notation, $\partial Y_F/\partial v = Y_v$, $\partial N_F/\partial r = N_r$ ... etc. These are known as the hydrodynamic derivatives. The physical meaning of these mathematical derivatives is very important to the development of understanding the connection between maneuvering and hydrodynamic forces. For example, $Y_r$ is the force in the sway direction due to a unit change in the yaw angular velocity.

Using this conventional notation, and substituting into equations (12), (13), and (14), the *linear* equations of motion in the horizontal plane in the absence of environmental disturbances (ideal handling conditions) and with the control surfaces set at zero, become:

$$(m - X_{\dot u})\,\dot u = X_u(u - U) \quad , \tag{27}$$

$$(m - Y_{\dot v})\,\dot v - (Y_{\dot r} - m x_G)\,\dot r = Y_v v + (Y_r - mU)\,r \quad , \tag{28}$$

$$(I_z - N_{\dot{r}})\,\dot{r} - (N_{\dot{v}} - mx_G)\,\dot{v} = N_v v - (N_r - mx_G U)\,r \quad . \tag{29}$$

where we can see that within linearity the surge equation decouples from sway and yaw.

Note that all of the terms of equations (28) and (29) _must_ include the effect caused by holding the vessel's rudder at zero in order for all the previously imposed simplifying conditions to hold true. If the possibility of non-controls fixed motion were to be considered, equations (28) and (29) must include terms on their respective right sides expressing the forces and moments caused by control surface deflection (i.e. rudder deflection) as a function of time. Assuming that the control surface most generally used in the horizontal plane is indeed a rudder and that the rudder force and moment on the vessel are functions of the rudder angle $\delta$ only, and not $\dot{\delta}$, the following is true:

$$X_R(\delta) = X_R(\delta=0) + \frac{\partial X_R}{\partial \delta}\delta = 0 \quad , \tag{30}$$

$$Y_R(\delta) = Y_R(\delta = 0) + \frac{\partial Y_R}{\partial \delta}\delta = Y_\delta \delta \quad , \tag{31}$$

$$N_R(\delta) = N_R(\delta = 0) + \frac{\partial N_R}{\partial \delta}\delta = N_\delta \delta \quad . \tag{32}$$

Positive rudder deflection corresponds to a port turn when associated with a stern rudder. $Y_\delta$ and $N_\delta$ are the rudder hydrodynamic derivatives. If the rudder forces and moments are now included in the linearized equations for sway and yaw (equations (28) and (29), respectively), the equations of motion become:

$$(m - Y_{\dot v})\,\dot v - (Y_{\dot r} - mx_G)\,\dot r = Y_v v + (Y_r - mU)\,r + Y_\delta \delta \quad , \qquad (33)$$

$$(I_z - N_{\dot r})\,\dot r - (N_{\dot v} - mx_G)\,\dot v = N_v v + (N_r - mx_G U)\,r + N_\delta \delta \quad . \qquad (34)$$

## 2. Stability of Straight Line Motion

The concept of an object remaining on a set path is related to the concept of a vessel's course stability or the stability of its straight line motion. There are various types of motion stability associated with marine vehicles and they are separated by the attributes of their initial equilibrium state that are retained in the final path of their center of gravity. These types of motion stability are depicted in Figure 1. For each type of stability depicted, the vessel is initially travelling at constant speed along a straight path. Case I, straight line or dynamic stability, retains the attribute of straight line motion but not the

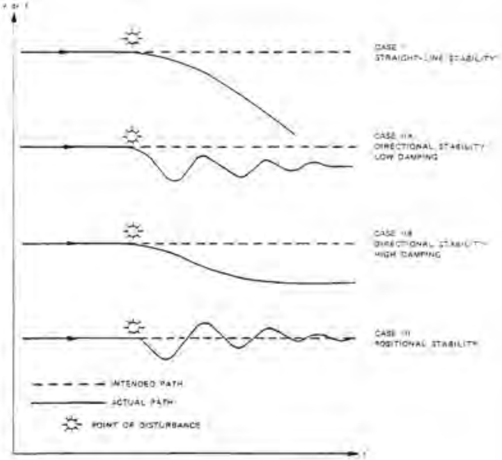


Figure 1: Various Kinds of Motion Stability

attribute of direction from the initial state of equilibrium after release from a disturbance. In Case II, *directional* stability, the final path after release from a disturbance retains not only the straight line attribute of the initial path, but also its direction. Case III is similar to Case II except that the vessel does not oscillate after the disturbance, but passes smoothly to the same path as in Case II. Finally, in Case IV, *positional* motion stability, the vessel returns to the original path and retains direction as well as position.

The types of stability are classified in an ascending order. A directionally stable vessel must also posses straight line stability. One that is positionally stable must posses both directional and straight line stability. Straight line stability or instability is indicated by the solution to a second order differential equation, directional stability or instability is indicated by the solution to a third order differential equation, and positional stability or instability is indicated by the solution to a fourth order differential equation.

Each type of controls fixed stability has an associated numerical index which by its sign determines the relative stability or instability of the ship. The magnitude of the index determines the degree of stability or instability. In order to illustrate the determination of this index, and to be able to evaluate the degree of dynamic stability, a return to

the differential equations of motion (28) and (29) is necessary. This homogeneous set of differential equations will have a solution of the form:

$$v(t) = v_1 e^{\sigma_1 t} + v_2 e^{\sigma_2 t} \; , \tag{35}$$

$$r(t) = r_1 e^{\sigma_1 t} + r_2 e^{\sigma_2 t} \; , \tag{36}$$

where the constants $v_1$, $v_2$, $r_1$, $r_2$, are determined from the initial conditions. Differentiation and subsequent substitution of equations (36) and (37) into equations (28) and (29) yields the following characteristic equation:

$$A\sigma^2 + B\sigma + C = 0 \; , \tag{37}$$

where

$$A = (I_z - N_{\dot{r}})(m - Y_{\dot{v}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G) \quad , \qquad (38)$$

$$B = -(I_z - N_{\dot{r}})Y_v - (m - Y_{\dot{v}})(N_r - mx_G U)$$

$$- (Y_r - mU)(N_{\dot{v}} - mx_G) - (Y_{\dot{r}} - mx_G)N_v \quad , \qquad (39)$$

$$C = (N_r - mx_G U)Y_v - (Y_r - mU)N_v \quad , \qquad (40)$$

with roots

$$\sigma_1, \sigma_2 = \frac{1}{2}\left[-\frac{B}{A} \pm \sqrt{\left(\frac{B}{A}\right)^2 - 4\left(\frac{C}{A}\right)}\ \right] \quad . \qquad (41)$$

If on course stability is to be displayed, then $\sigma_1$ and $\sigma_2$ will have negative real parts. Referring back to equations (36) and (37), both v(t) and r(t) will go to zero for increasing time. This means that the ship will eventually assume a straight line direction at a different, in general heading. This is straight line stability as defined in Figure 1. Symbolically, the stability requirement

$$\Re(\sigma_1, \sigma_2) < 0 \ ,$$

translates to

$$A > 0, \quad B > 0, \quad C > 0 \, .$$

with all the coefficients of the quadratic equation (58) are positive. To evaluate these conditions, we must check the signs of the hydrodynamic derivatives that make up A, B and C. A physical interpretation of the hydrodynamic variables as forces and moments acting on the hull due to the various hull motions with respect to the water is necessary.

After some analysis, the following conclusions are drawn:

$Y_{\dot v}$ is always negative and large.

$Y_v$ is always negative and large.

$N_{\dot r}$ is always negative and large.

$N_r$ is always negative and large.

$Y_{\dot r}$ is small and of uncertain sign.

$Y_r$ is small and of uncertain sign.

$N_{\dot v}$ is small and of uncertain sign.

$N_v$ is small and of uncertain sign.

$x_G$ is small and of uncertain sign.

$I_z$ is always positive and large.

$m$ is always positive and large.

With these conditions described thusly, A and B are therefore always positive and stability must require:

$$C = (N_r - mx_G U)\, Y_v - (Y_r - mU)\, N_v > 0 \quad . \tag{42}$$

This is the Stability Criterion. The more positive C is, the more stable the ship will become and the more increasingly difficult the ship will be to maneuver. The more negative C is, the more unstable the ship will become and the continuous use of rudder will be required to maintain course. Although $x_G$ is small and of uncertain sign, for stability it is desired to have $x_G > 0$ (center of gravity forward of amidships). Standard ship design experience agrees with this observation.

## B. MANEUVERING IN THE VERTICAL PLANE

### 1. Linear Equations in the Vertical Plane

Maneuvers in the vertical plane can be describe in much the same manner as they can in the horizontal plane. The logical progression in the determination of stability will be traced out here for the vertical plane as it was in the previous section for the horizontal plane.

A submerged vessel moving in the vertical plane has a nominal path defined as the level path at the commanded depth. It has a pitch angle, $\theta$, which is defined as the angle between the vehicle's longitudinal axis and the x axis and an angle of attack, $\alpha$, which is defined as the angle between the x axis and the total velocity vector. Figure 2 illustrates these points. If Newton's law of motion is once again used, the

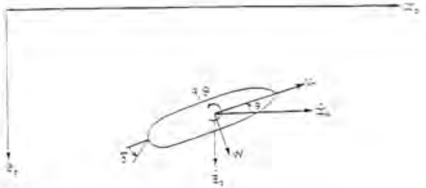


Figure 2: Vehicle Geometry in the Vertical Plane

following equations are true using the variables introduced in Chapter I:

$$m\ddot{x}_0 = X_0 \quad , \tag{43}$$

$$m\ddot{z}_0 = Z_0 \quad , \tag{44}$$

$$I_y\ddot{\theta} = M \quad , \tag{45}$$

As in the horizontal plane, the vertical transformation of the coordinate system yields:

$$
\begin{aligned}
X_0 &= +X\cos\theta + Z\sin\theta \;, \\
Z_0 &= -X\sin\theta + Z\cos\theta \;, \\
\dot{x}_0 &= +u\cos\theta + w\sin\theta \;, \\
\dot{z}_0 &= -u\sin\theta + w\cos\theta \;,
\end{aligned}
\qquad (46)
$$

where the equations of motion are:

$$
\begin{aligned}
m\dot{u} + mwq &= X \;, \quad \textit{surge equation} \;, \\
m\dot{w} - muq &= Z \;, \quad \textit{heave equation} \;, \\
I_y\dot{q} &= M \;, \quad \textit{pitch equation} \;.
\end{aligned}
\qquad (47)
$$

and

$$
\dot{\theta} = q \;. \qquad (48)
$$

When expressed with respect to the vessel's reference frame, these equations become:

$$
m\dot{u} + mwq - mx_G q^2 + mz_G\dot{q} = X \;, \qquad (49)
$$

$$m\dot{w} - muq - mx_G\dot{q} - mz_Gq^2 = Z \quad . \tag{50}$$

$$I_y\dot{q} - mx_G(\dot{w} - uq) + mz_G(\dot{u} - wq) = M \quad . \tag{51}$$

where $(x_G, z_G)$ are the coordinates of the ship's center of gravity with respect to the ship fixed reference frame, and we assume $y_G = 0$.

Equations (49) through (51) demonstrate a coupling between surge motion and vertical plane motion that was not present in the discussion on the horizontal plane. If the vertical plane equations of motion are linearized around a level flight path at constant speed, $U$, we can see from equations (49) and (51) that the two terms $mz_G\dot{q}$ and $mz_G\dot{u}$ will remain after linearization. This means that heave and pitch are coupled to surge even for small motions. This dynamic coupling arises due to the nonzero $z_G$ term. When a vessel pitches, a nonzero surge acceleration will contribute to the inertial moment. Obviously there was no such linearized dynamic coupling in the horizontal plane. Although vertical plane motion must be studied together with surge, the coupling discussed here is relatively small due to small values of $z_G$ and in this case can be neglected. Therefore, the uncoupled linearized equations of motion in the vertical plane are:

$$m\dot{w} - mUq - mx_G\dot{q} = Z \quad . \tag{52}$$

$$I_y\dot{q} - m x_G(\dot{w} - Uq) = M \quad . \tag{53}$$

Much like in the previous section, Z and M can be expressed in terms of slow motion derivatives. Using vertical plane notation they are:

$$Z = Z_w w + Z_q q + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_\delta \delta \quad , \tag{54}$$

$$M = M_w w + M_q q + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} - (z_G - z_B) W \sin\theta + M_\delta \delta \quad , \tag{55}$$

where $Z_w$ stands for the change in the hydrodynamic force in Z with a unit change in w (note: $Z_w$ is synonymous to $Y_v$ in the horizontal plane) and the same for the other variables. The additional term in equation (55) is the hydrodynamic restoring moment, and all other variables are in accordance with the listing given in Chapter I. Provided the vessel is properly balanced (weight = buoyancy), there is no such hydrostatic force in the heave equation (54). The $\delta$ in these equations refers to the stern plane angular deflection as shown in Figure 2.

Equations (52) through (55) lead to the linear uncoupled equations of motion in the vertical plane:

$$(m - Z_{\dot{w}})\dot{w} - (m x_G + Z_{\dot{q}})\dot{q} = Z_w w + (Z_q + mU) q + Z_\delta \delta \quad , \tag{56}$$

$$(I_y - M_{\dot q})\dot q - (m x_G + M_{\dot w})\dot w = M_w w + (M_q - m x_G U) q - (z_G - z_B) W \theta + M_\delta \delta \tag{57}$$

These two equations can be used with equation (48) in order to predict the response of the vessel in the vicinity of a nominal level flight path. Deviations in depth can be computed from:

$$\dot z = -U\sin\theta + w\cos\theta \; , \tag{58}$$

or

$$\dot z = -U\,\theta \tag{59}$$

in its linearized form.

**2. Stability of Motion**

Stability of motion in the vertical plane can be studied by using equations (56) and (57) after $\delta$ is set to zero. As in the case of the horizontal plane, small motion are being studied around the nominal flight path with the controls fixed. Equations (56) and (57) become:

$$(m - Z_{\dot w})\dot w - (mx_G + Z_{\dot q})\dot q - Z_w w - (m + Z_q) q = 0 \; , \qquad (60)$$

$$(I_y - M_{\dot q})\dot q - (mx_G + M_{\dot w})\dot w - M_w w - (M_q - mx_G) q - M_\theta \theta = 0 \; , \qquad (61)$$

where

$$M_\theta = -(z_G - z_B) W = -(z_G - z_B) mg \; . \qquad (62)$$

and $U = 1$ since dimensionless units are commonly used. The $M_\theta$ term is the hydrostatic 'spring constant' due to the nonzero metacentric height. The corresponding term in the horizontal plane is $N_\psi$ and is equal to zero. The negative sign in equation (62) makes $M_\theta < 0$, which is consistent with established definitions for positive forces and moments in the right handed orthogonal coordinate system. Manipulating equations (60), (61), and (46) and transforming to the s domain gives the following:

$$[(m - Z_{\dot w}) s - Z_w] w = [-(Z_{\dot q} + mx_G) s^2 - (Z_q + m) s]\theta \; , \qquad (63)$$

$$[-(M_{\dot w} + mx_G) s - M_w] w + [(I_y - M_{\dot q}) s^2 - (M_q - mx_G) s - M_\theta]\theta = 0 \; ; \qquad (64)$$

where the characteristic equation is:

$$A\sigma^3 + B\sigma^2 + C\sigma + D = 0 \quad , \tag{65}$$

where,

$$A = (m - Z_w)(I_y - M_q) - (M_w + mx_G)(Z_q + mx_G) \quad . \tag{66}$$

$$B = -(m - Z_w)(M_q - mx_G) - (Z_q + mx_G)M_w - (Z_q + m)(M_w + mx_G)$$

$$-(I_y - M_q)Z_w \quad , \tag{67}$$

$$C = (M_q - mx_G)Z_w - (Z_q + m)M_w - (m - Z_w)M_\theta \quad , \tag{68}$$

$$D = Z_w M_\theta \quad . \tag{69}$$

When comparing equations (66) through (69) with equations (38) through (40), coefficients A through C are similar in the horizontal and vertical planes with the extra metacentric moment term appearing in equations (68) and (69). If there were no hydrostatic terms, this cubic characteristic equation (65) would reduce to a quadratic similar to the equation derived in the horizontal plane. This means that there is a zero root in this case. A physical analysis is necessary to

understand the full meaning of this mathematical fact. In the vertical plane:

$$(As^3 + Bs^2 + Cs + D)\theta = 0 \tag{70}$$

This means that, assuming certain conditions of stability are satisfied, it is possible for $\theta(t) \rightarrow 0$ as $t \rightarrow \infty$. Therefore, directional stability is possible. Where in the horizontal plane,

$$(As^2 + Bs + C)s\psi = (As^2 + Bs + C)\dot{\psi} = 0 \tag{71}$$

This means that the best that can be expected in the horizontal plane is $\dot{\psi}(t) = r(t) \rightarrow 0$ as $t \rightarrow \infty$, which means that $\psi$ will be increasing linearly with time. Therefore, directional stability is not possible and the best possible situation is straight line stability.

In the case of the cubic equation the Routh-Hurit conditions for stability dictate that all coefficients A, B, C, and D be positive and that:

$$BC - AD > 0 \quad . \tag{72}$$

After close analysis similar to that conducted in the horizontal plane, it can be proven that A, B and D are positive. Therefore in order for the vessel to stable in the vertical plane:

$$C = (M_q - mx_G)Z_w - (Z_q + m)M_w - (m - Z_{\dot{w}})M_\theta > 0 \quad . \tag{73}$$

Since $(m - Z_{\dot{w}}) > 0$ and $M_\theta < 0$, this condition is easier to satisfy in the vertical plane than in the horizontal plane. This proves that a positive metacentric height contributes to dynamic stability as well as static stability. Assuming that equation (73) is satisfied, it can be shown that equation (72) is always satisfied. This is best shown by using a method detailed in [Ref.3]. If the coefficients are written as:

$$A = A_0$$
$$B = B_0$$
$$C = C_0 + \epsilon C_1$$
$$D = \epsilon D1$$

where the subscript 0 refers to the hydrodynamic coefficients without the hydrostatic terms, and subscript 1 refers to the

metacentric terms. The parameter $\epsilon$ is the metacentric height $(z_G - z_B)$. Therefore:

$$BC - AD = B_0 C_0 + \epsilon (B_0 C_1 - A_0 D_1) \qquad (74)$$

Based on the given conditions, $B_0$, $C_1$, $A_0$, and $D_1$ are all positive. Therefore the difference $(B_0 C_1 - A_0 D_1)$ is a small number of uncertain sign. Since it is multiplied by $\epsilon$, a small positive number, the result is a small number of uncertain sign. This small number, regardless of sign will not change the sign in equation (74), assuming that $C_0$ is positive. This means that $C_0 > 0$ will be a sufficient condition to satisfy vertical plane stability. The vertical plane stability reduces to:

$$(M_q - m x_G) Z_w - (Z_q + m) M_w > 0 \qquad (75)$$

Where this is a sufficient condition for vertical stability, it is not a necessary condition. It is possible that the hydrodynamic term will stabilize the vessel and equation (72) will be satisfied (and all coefficients will be positive), even though equation (75) might not hold true. This condition is dependent on the magnitude of the metacentric height and

the speed of the vessel. As a vessel experiences an increase in forward speed, it will behave less stably in the vertical plane.

Recalling the Stability Criterion for the horizontal plane from equation (42), equation (75) can serve as the Stability Criterion for the vertical plane. These two expressions are commonly normalized into *Stability Indices*:

$$G_v = 1 - \frac{M_w(Z_q + m)}{Z_w(M_q - mx_G)} \quad , \tag{76}$$

$$G_h = 1 - \frac{N_v(Y_r - m)}{Y_v(N_r - mx_G)} \quad . \tag{77}$$

These two indices are the vertical and horizontal plane stability indices and they are used to asses maneuverability and relative stability. They are always less than one, and positive values indicate stability and negative values indicate instability.

## III. MANEUVERING IN SIX DEGREES OF FREEDOM

### A. RIGID BODY EQUATIONS OF MOTION AND KINEMATICS

Development of the general maneuvering equations in six degrees of freedom requires the employment of two coordinate systems depicted in Figure 3:

O(X,Y,Z) : reference inertial frame.

A(x,y,z) : moving frame fixed on the vessel's geometric center.

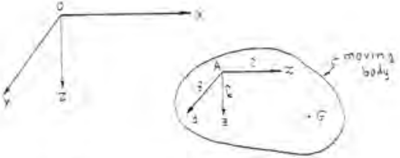


**Figure 3:** Fixed and Moving Coordinate Systems

As discussed in the previous chapter, Newton's law requires an inertial frame, while the motion of the vessel is best described in its own frame. G denotes the center of gravity of the moving body, which is located at $(x_G, y_G, z_G)$ with respect to the A(x,y,z) frame. Unit vectors in this moving frame are denoted by $\hat{i}, \hat{j}, \hat{k}$, in the x,y,z directions, respectively.

The translational equations of motion can be found through the use of the principle of linear momentum in the inertial frame. The rate of change of the linear momentum equals the sum of the external forces. Vector analysis is then used to express these quantities in the A system. Chapter 4.1 of Reference 3 explains this conversion in detail. Once the conversion to the A system is accomplished, Newton's law can be broken into components as:

$$m[\dot{u} + qw - rv - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X \ , \tag{78}$$

$$m[\dot{v} + ru - pw - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] = Y \ , \tag{79}$$

$$m[\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z \ . \tag{80}$$

where the total exciting force ,F, is denoted by,

$$F = X\hat{i} + Y\hat{j} + Z\hat{k} \quad . \tag{81}$$

Therefore the external moment, M can be written as:

$$M = K\hat{i} + M\hat{j} + N\hat{k} \quad . \tag{82}$$

and broken into components as:

$$I_x \dot{p} + (I_z - I_y)qr + I_{xy}(pr - \dot{q}) - I_{yz}(q^2 - r^2) - I_{xz}(pq + \dot{r})$$

$$+ m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} + ur - wp)] = K \quad , \tag{83}$$

$$I_y \dot{q} + (I_x - I_z)pr - I_{xy}(qr + \dot{p}) + I_{yz}(pq - \dot{r}) + I_{xz}(p^2 - r^2)$$

$$-m[x_G(\dot{w} - uq + vp) - z_G(\dot{u} - vr + wq)] = M \quad , \tag{84}$$

$$I_z \dot{r} + (I_y - I_x)pq - I_{xy}(p^2 - q^2) - I_{yz}(pr + \dot{q}) + I_{xz}(qr - \dot{p})$$

$$+m[x_G(\dot{v} + ur - wp) - y_G(\dot{u} - vr + wq)] = N \quad . \tag{85}$$

Equations (78) through (80) are the translational equations of motion, and together with the rotational equations (83) through (85), they make up the six degrees of freedom of motion of a rigid body expressed in a coordinate system moving

along with the body. Once again the variables involved are the same variables introduced in Chapter I.

Although the equations of motion are written in a frame associated with the vehicle, the vessel's orientation and position in the inertial frame is still very important. The angular (orientational relationship between the two frames is determined by the three Euler angles:

| | | |
|---|---|---|
| yaw or heading, $\psi$ | : | rotation about x axis |
| pitch or elevation, $\theta$ | : | rotation about y axis |
| roll or bank, $\phi$ | : | rotation about z axis |

Any vector described in the two reference frames are related by some transformation matrix, $T$ which is a function of these three Euler angles.

Therefore,

$$T = T(\phi, \theta, \psi) \quad . \tag{86}$$

The general transformation matrix can be broken up into three individual rotations. The order of application is very critical in order to maintain the integrity of the

transformation. Physically, infinite rotations do not commute. The prescribed manner is to first yaw, then pitch, and finally to roll. Standard matrix notation therefore yields:

$$T(\phi,\theta,\psi) = T(\phi)\,T(\theta)\,T(\psi) \quad . \tag{87}$$

The full transformation matrix is:

$$T(\phi,\theta,\psi) = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi\cos\phi+\sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi+\cos\phi\cos\psi\sin\theta & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \tag{88}$$

Small rotations do commute, and for small angles,

$$T(\phi,\theta,\psi) = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \quad . \tag{89}$$

and in this case $T(\phi,\theta,\psi) = T(\theta,\phi,\psi)$. Equations (87) and (88) can now be used to transform any vector in one system to its

equivalent in the other system through the use of matrix multiplication. Here are some important examples of the relationship between the two reference frames:

$$\dot{x}_o = u\cos\theta\cos\psi + v(-\sin\psi\cos\phi + \sin\phi\sin\theta\cos\psi)$$

$$+w(\sin\theta\sin\psi + \cos\phi\cos\psi\sin\theta) \quad , \qquad (90)$$

$$\dot{y}_o = u\cos\theta\sin\psi + v(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi)$$

$$+w(-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi) \quad , \qquad (91)$$

$$\dot{z}_o = -u\sin\theta + v\sin\phi\cos\theta + w\cos\phi\cos\theta \quad , \qquad (92)$$

$$p = -\dot{\psi}\sin\theta + \dot{\phi} \quad , \qquad (93)$$

$$q = \dot{\psi}\sin\phi\cos\theta + \dot{\theta}\cos\phi \quad , \qquad (94)$$

$$r = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \quad , \qquad (95)$$

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \quad , \qquad (96)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \quad , \qquad (97)$$

$$\dot{\psi} = q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta} \quad , \qquad (98)$$

where the subscript o represents the inertial reference frame. For small angles, equations (96) through (98) become:

$$\dot{\phi} = p\,,\ \dot{\theta} = q\,,\ \dot{\psi} = r\ ,$$

which means that the rates of change of the Euler angles are the same as the angular velocities *only* for infinitesimal rotations. The six kinematic equations (90) through (92) and (96) through (98) along with equations (78) through (80) and (83) through (85) provide a complete description of the body's motion in six degrees of freedom.

**B. EXTERNAL FORCES AND MOMENTS**

Equations (78) through (80) and (83) through (85) are the six degrees of freedom equations of motion for a rigid body where the left hand terms represent inertial forces and moments while the right hand terms represent the external forces and moments. These external forces and moments include hydrodynamic contributions as well as weight and buoyancy effects and forces from control surface deflections and propeller thrust. Hydrodynamic restoring forces and moments are due to the vehicle weight and buoyancy. Forces and moments due to control surface deflections are experienced as added drag in surge, while in sway, heave, pitch, and yaw they are directly proportional to the deflection. Just as the position and velocity vectors in the previous section can be

expressed in terms of either the inertial reference frame or the vessel's relative reference frame, so can these forces and moments. The tie between the two frames of reference in, once again the transformation matrix. Particularly in the case of the hydrodynamic force and moment contributions, the derivation of these binding expressions can be very complicated and lengthy. If a more in depth discussion of the subject is desired, Reference 3 provides an excellent overview.

## C. LINEARIZATION

Linearization, as taught as a mathematical tool is useful for understanding the behavior of nonlinear systems. The principle is used here to analyze particular vehicle nonlinear motion about a nominal flight path. Generally, linearization is performed in the vicinity of a nominal point. This point is that point where the system is expected to spend most of its life. The nominal point is defined by the system condition that all time derivatives equal zero. Any existing system controls can be fixed or expressed as a function of the nominal point. Once a system solution has been computed using the nominal point and the necessary qualifing condition, the system can be linearized. This is done by determining the Jacobian matrices of the system. This new linearized system can be used to analyze system open loop stability through the examination of the eigenvalues of the Jacobian.

## IV. DISCUSSION AND RESULTS

### A. GENERAL

The major motivation of this thesis is to further explore the line of reasoning that underwater vehicles are no longer bound by the straight line motion that is caused by personnel or cargo staged onboard the vessel. This thesis seeks to prove/verify that a particular underwater vehicle is stable/unstable when enough physical information is known. While gathering information to form the stability analysis, valuable information can be obtained concerning the motion of the vehicle while it performs three dimensional maneuvers. The objective of this study is to provide a parametric stability analysis of the Swimmer Delivery Vehicle in six degree of freedom motion. All physical properties of the vehicle are known as well as the linear, nonlinear and coupling terms. The main object is to asses control design while keeping the controls fixed and varying the distance between the longitudinal center of gravity (LCG) and the longitudinal of buoyancy (LCB).

### B. PROCEDURE

After a prerequisite understanding of uncoupled motion in the horizontal and vertical plane, the general procedure followed in this parametric study continued with the full six

degree of freedom equations of motion. The necessary variables in order to analyze the system dynamics are the translational velocities surge, sway, and heave (u, v, w); rotational velocities roll, pitch, and yaw (p, q, r); the Euler angles of roll and pitch ($\phi$, $\theta$); and the rate of change of heading - the first derivative of the yaw angle, $\dot{\psi}$. In order to provide a comprehensive analysis of vehicle behavior and capability, a wide variety of realistic control parameters and loading conditions were chosen and evaluated for their contribution to the steady state motions of the above stated variables. The final step in the procedure is to linearize in the vicinity of the steady state and to compute the system eigenvalues. These eigenvalues serve as indicators of the relative stability of the vehicle under the given conditions.

## C. STEADY STATE ANALYSIS

The steady state analysis started with the full, nonlinear, coupled six degree of freedom equations of motion. All time derivatives were set to zero since steady state is defined as that point where there is no change in the state of a variable with respect to time. The resulting algebraic system of coupled, nonlinear equations was solved for the control parameters and loading conditions used. In order to perform an adequate analysis, a continuation algorithm developed by *Seydel* (Ref.1), with subroutines by Aydin (Ref.2)

were used. The following range of control parameters were examined:

- stern diving plane angle varied between ±20 degrees
- rudder angle varied between 0 and 20 degrees
- LCG/LCB separation between ±1.5 percent of vehicle length

The graphical results are organized in sets according to LCG/LCB separation. The LCG/LCB separation was varied between ±1.5 percent of vehicle length in increments of 0.5 percent. Each set of the following continuation results depicts the entire stern diving plane angle range as the x axis. Each set has a plot where the y axis is one of the aforementioned variables u, v, w, p, q, r, $\phi$, $\theta$, and $\psi$. The translational velocities are plotted in ft/sec, and the angular velocities in rad/sec. All simulations were run at a constant propeller speed of 500 rpm, which translates to a value of 6.0 ft/sec for the forward speed, u. The final plot in each set is the relative *degree of stability* which will be discussed in the next section. These plots must be analyzed as a set in order to provide a comprehensive understanding of the vehicle six degree of freedom motion. Through the comparison of these plots, the motion associated with any maneuver within the range of parameters can be predicted.

## D. STABILITY ANALYSIS

Steady state analysis of the maneuver alone is not sufficient. It must be determined if the vehicle is to remain stable (controllable) while the maneuver is negotiated. Therefore the final plot of the set depicts the degree of stability plotted against the range of the stern diving plane angle. The units of the degree of stability are in terms of inverse time, $sec^{-1}$. This index is obtained by evaluating the 9 x 9 Jacobian matrix created by the determination of the steady states discussed in the previous section. The largest real part of all resulting eigenvalues in a continuation is the degree of stability. This, in physical terms, is the most dominant time constant for the system. The index measures the slowest exponential convergence to the steady state motion when negative or the fastest exponential divergence from the steady state motion when positive. The more negative the index, the more stable the vehicle maneuver. Conversely, the more positive the index, the more unstable the maneuver.

## E. GRAPHICAL RESULTS

The following figures along with an understanding of the previous sections of this chapter illustrate to the reader the dynamics of vehicle motion and the stability of that motion within the parameters inputted into the continuation algorithm.

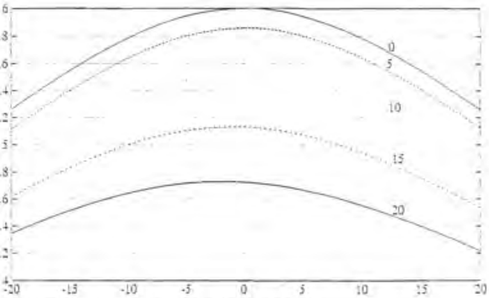


**Figure 4**: u vs Stern Plane Angle; $x_{cg}$= 0%

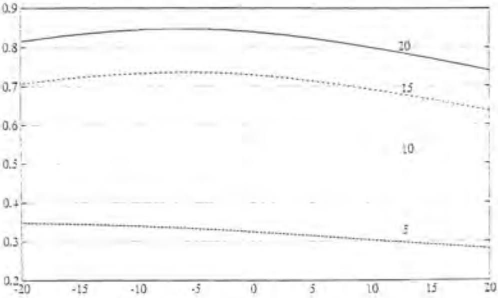


**Figure 5**: v vs Stern Plane Angle; $x_{cg}$= 0%

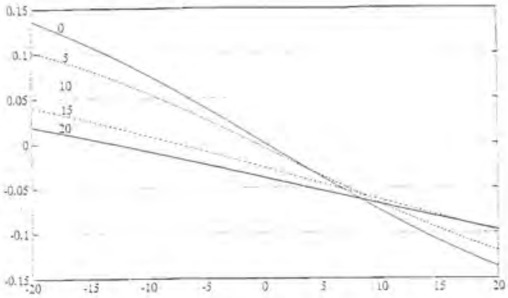


**Figure 6**: w vs Stern Plane Angle: $x_{cg}$= 0%

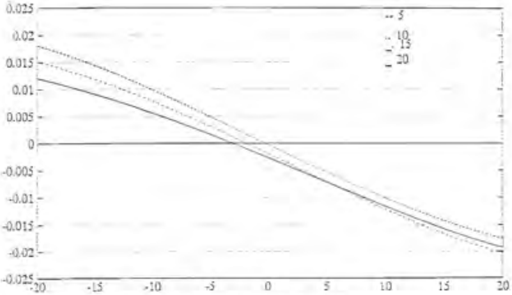


**Figure 7**: p vs Stern Plane Angle: $x_{cg}$= 0%

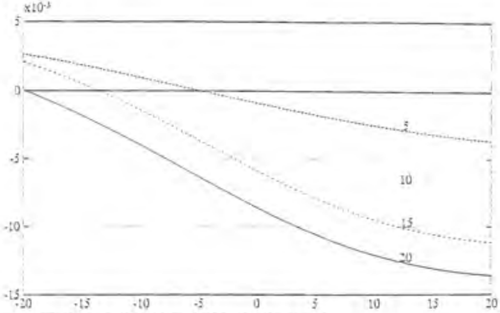


**Figure 8**: q vs Stern Plane Angle; $x_{cg}$= 0%

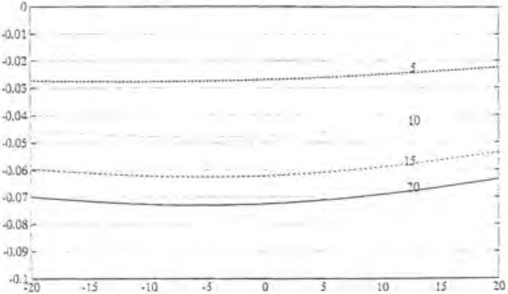


**Figure 9**: r vs Stern Plane Angle; $x_{cg}$= 0%

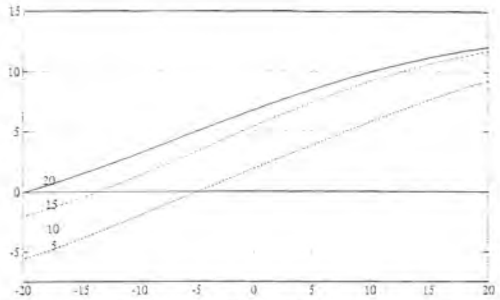


**Figure 10**: $\phi$ vs Stern Plane Angle: $x_{cg}$= 0%

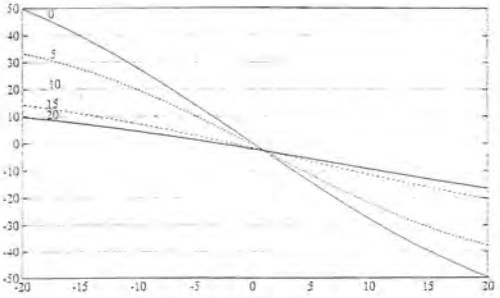


**Figure 11**: $\theta$ vs Stern Plane Angle: $x_{cg}$= 0%

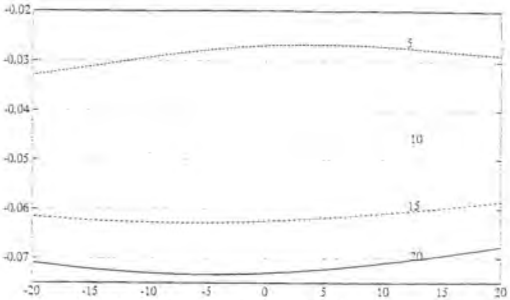


**Figure 12**: $\psi$ vs Stern Plane Angle; $x_{cg}$= 0%

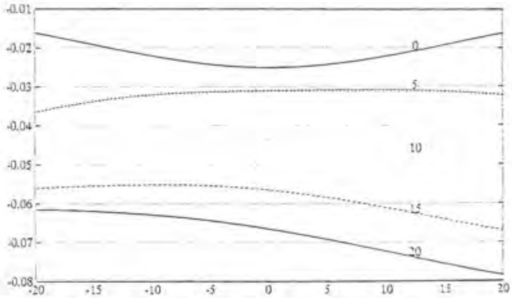


**Figure 13**: Stability Index vs Stern Plane Angle: $x_{cg}$= 0%

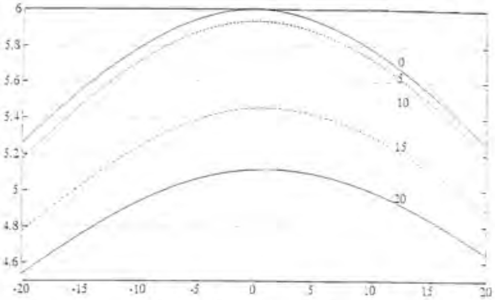


**Figure 14**: u vs Stern Plane Angle; $x_{cg}$=.5%

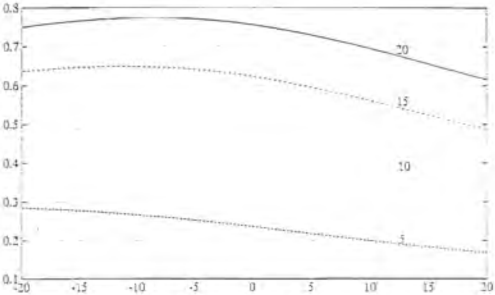


**Figure 15**: v vs Stern Plane Angle; $x_{cg}$=.5%

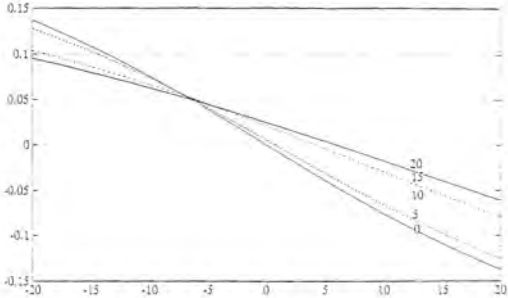


**Figure 16**: w vs Stern Plane Angle: $x_{cg}$=.5%

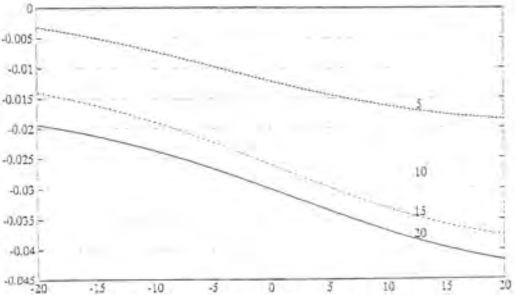


**Figure 17**: p vs Stern Plane Angle: $x_{cg}$=.5%

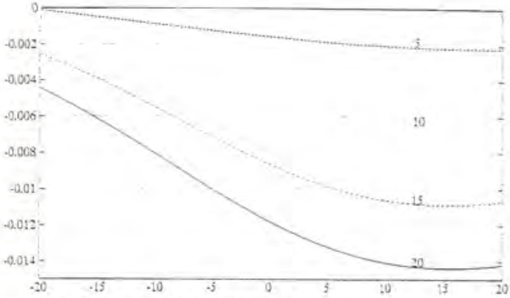


**Figure 18**: q vs Stern Plane Angle: $x_{cg}$=.5%

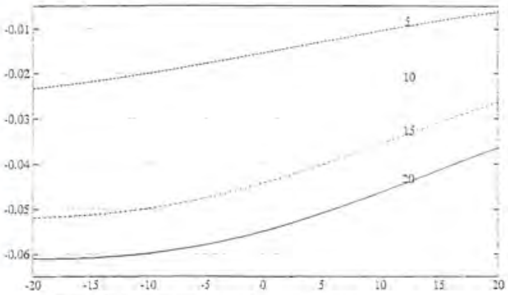


**Figure 19**: r vs Stern Plane Angle: $x_{cg}$=.5%

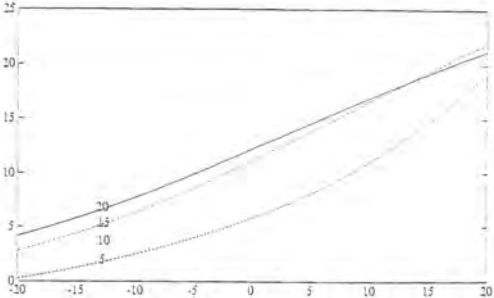


**Figure 20**: $\phi$ vs Stern Plane Angle; $x_{cg}$=.5%

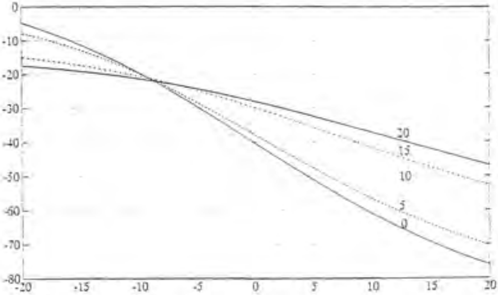


**Figure 21**: $\theta$ vs Stern Plane Angle; $x_{cg}$=.5%

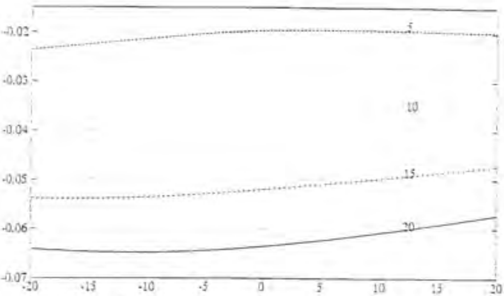


**Figure 22**: Φ vs Stern Plane Angle: $x_{ig}$=.5%

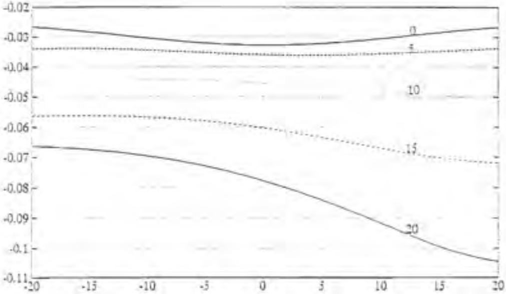


**Figure 23**: Stability Index vs Stern Plane Angle: $x_{ig}$=.5%

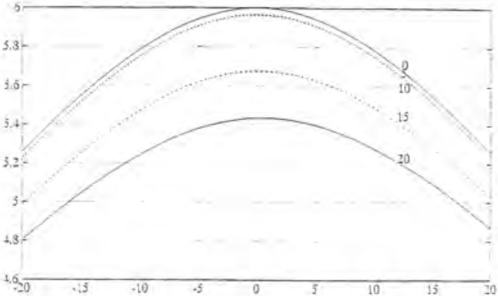


**Figure 24**: u vs Stern Plane Angle; $x_{cg}$=1.0%

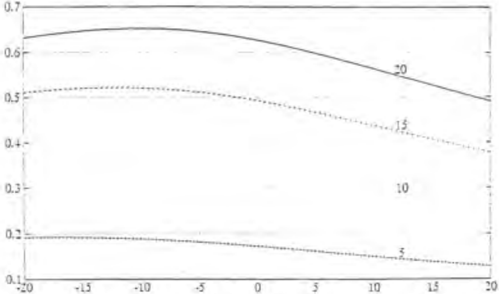


**Figure 25**: v vs Stern Plane Angle; $x_{cg}$=1.0%

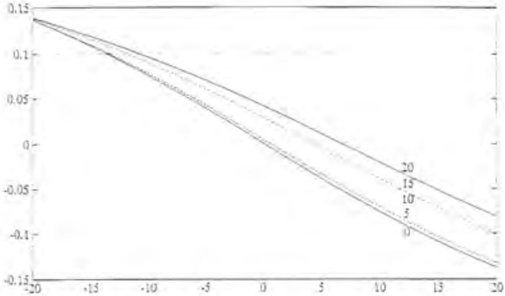


**Figure 26**: w vs Stern Plane Angle: $x_{CB}$=1.0%

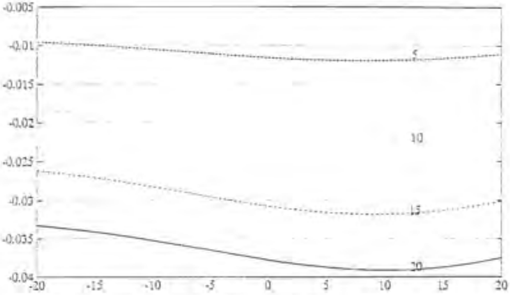


**Figure 27**: p vs Stern Plane Angle: $x_{CB}$=1.0%

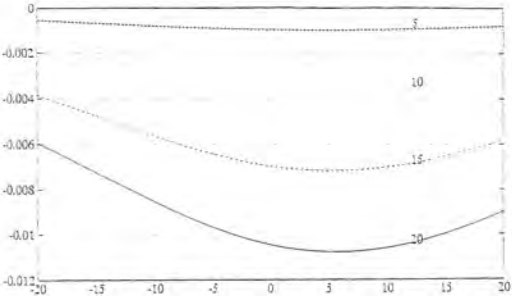


**Figure 28**: q vs Stern Plane Angle: $x_{cg}$=1.0%

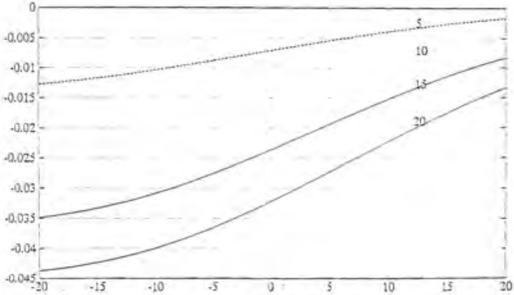


**Figure 29**: r vs Stern Plane Angle: $x_{cg}$=1.0%

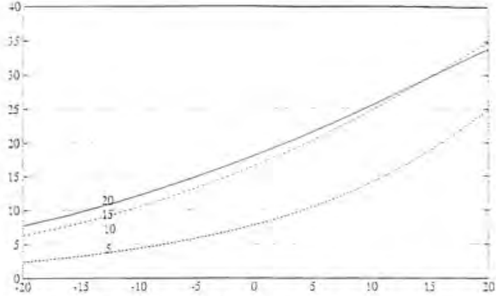


**Figure 30**: $\phi$ vs Stern Plane Angle; $x_{cg}$=1.0%

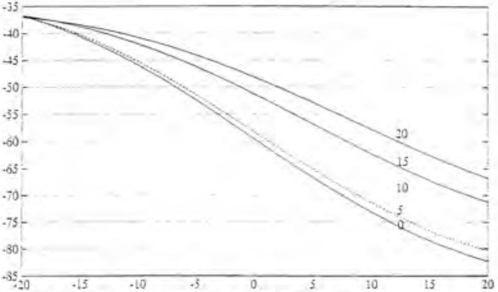


**Figure 31**: $\theta$ vs Stern Plane Angle; $x_{cg}$=1.0%

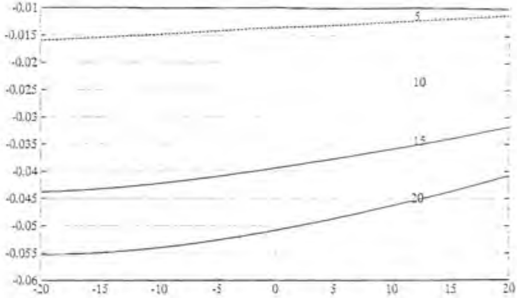


**Figure 32**: $\dot{\psi}$ vs Stern Plane Angle: $x_{cg}$=1.0%

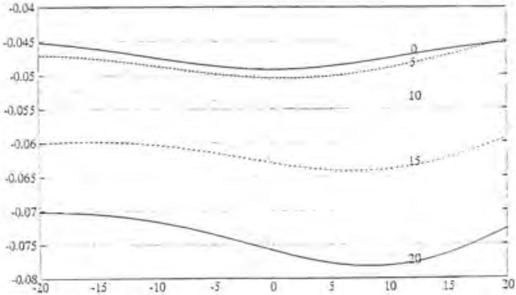


**Figure 33**: Stability Index vs Stern Plane Angle: $x_{cg}$=1.0%

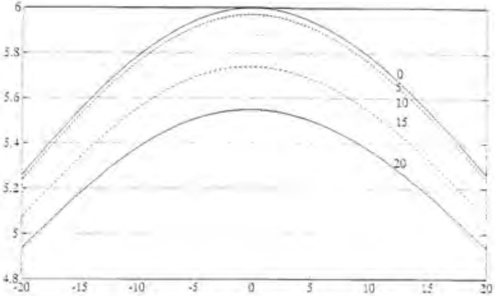


**Figure 34**: u vs Stern Plane Angle; $x_{cB}$=1.5%

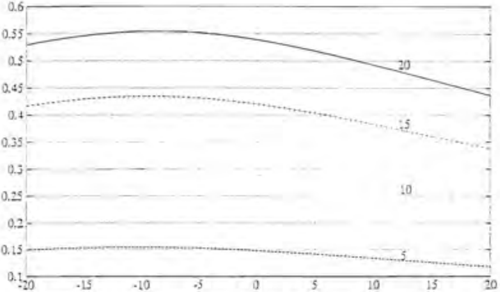


**Figure 35**: v vs Stern Plane Angle; $x_{cB}$=1.5%

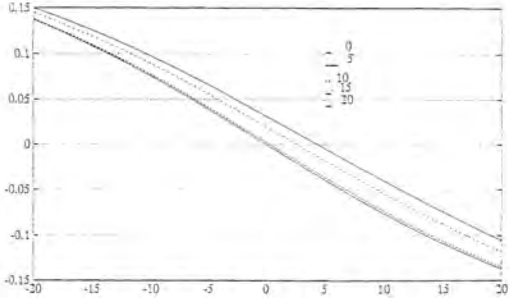


**Figure 36**: w vs Stern Plane Angle: $x_{cg}$=1.5%

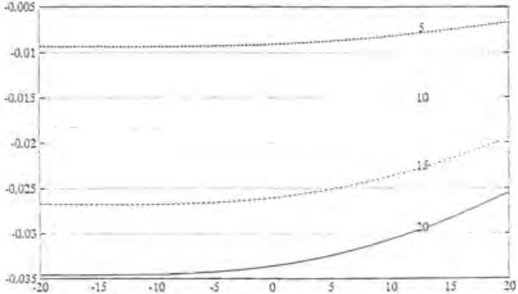


**Figure 37**: p vs Stern Plane Angle: $x_{cg}$=1.5%

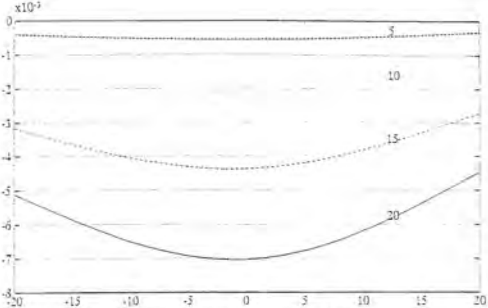


**Figure 38**: q vs Stern Plane Angle; $x_{cg}$=1.5%

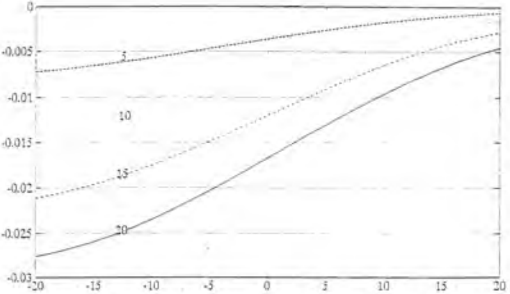


**Figure 39**: z vs Stern Plane Angle; $x_{cg}$=1.5%

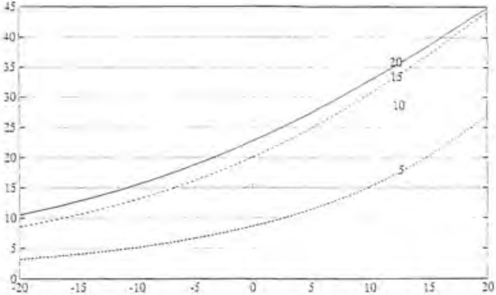


**Figure 40**: $\phi$ vs Stern Plane Angle; $x_{cw}$=1.5%

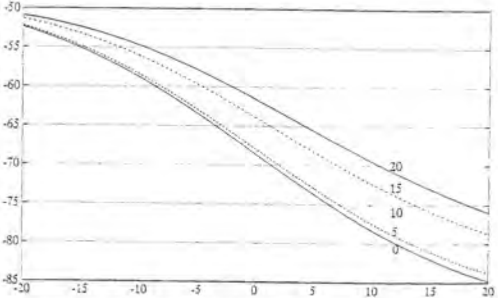


**Figure 41**: $\theta$ vs Stern Plane Angle; $x_{cw}$=1.5%

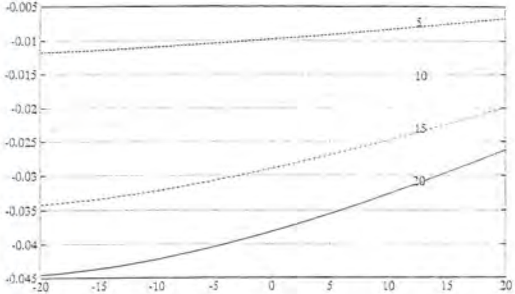


**Figure 42:** $\phi$ vs Stern Plane Angle: $x_{cg}$=1.5%

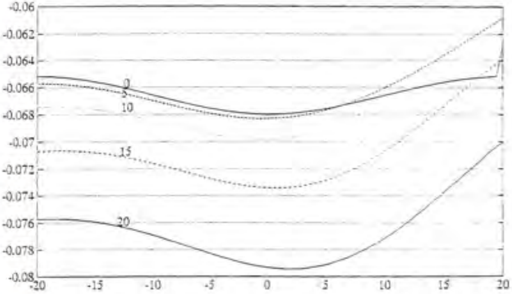


**Figure 43:** Stability Index vs Stern Plane Angle: $x_{cg}$=1.5%

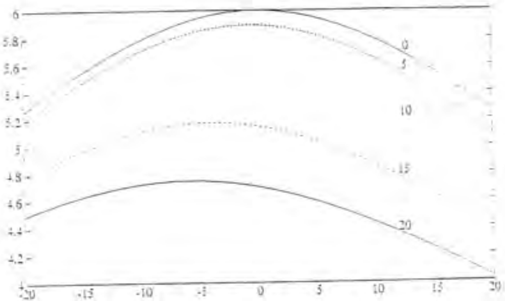

**Figure 44**: u vs Stern Plane Angle: $x_{cg}$=-.5%

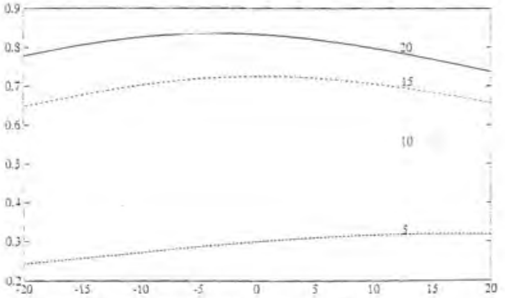

**Figure 45**: v vs Stern Plane Angle: $x_{cg}$=-.5%

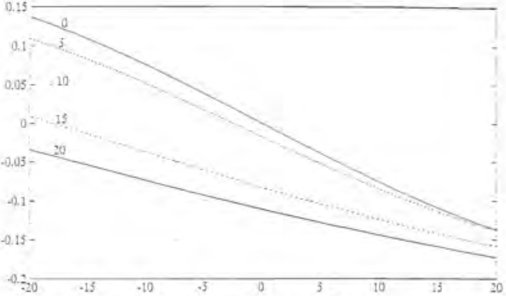


**Figure 46**: w vs Stern Plane Angle; $x_{cg}$=-.5%

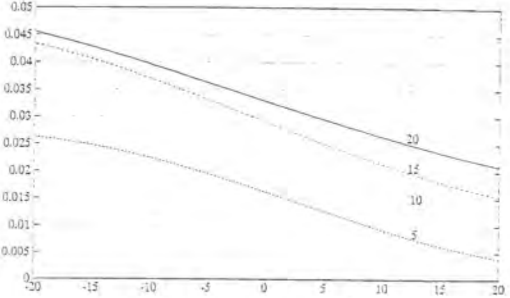


**Figure 47**: p vs Stern Plane Angle; $x_{cg}$=-.5%

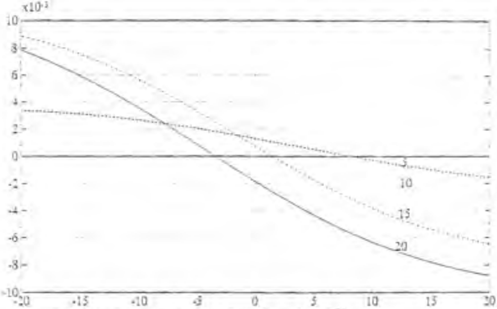

**Figure 48**: q vs Stern Plane Angle; $x_{cg}$=-.5%

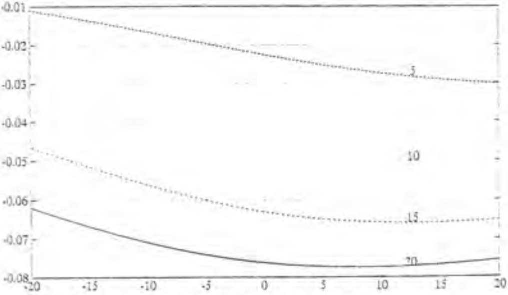

**Figure 49**: r vs Stern Plane Angle; $x_{cg}$=-.5%

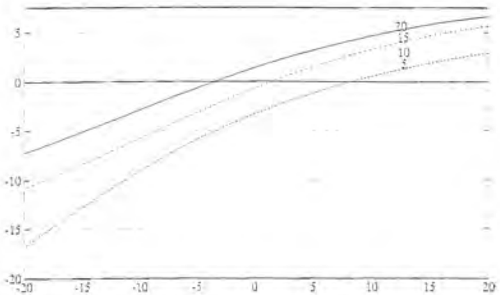


**Figure 50**: $\phi$ vs Stern Plane Angle; $x_{cg}$=-.5%

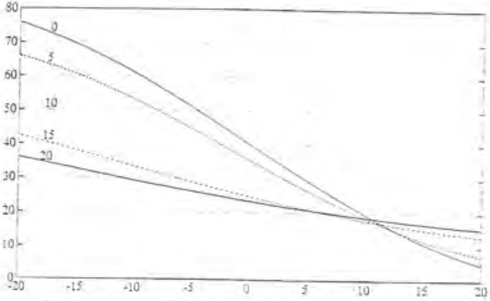


**Figure 51**: $\theta$ vs Stern Plane Angle; $x_{cg}$=-.5%

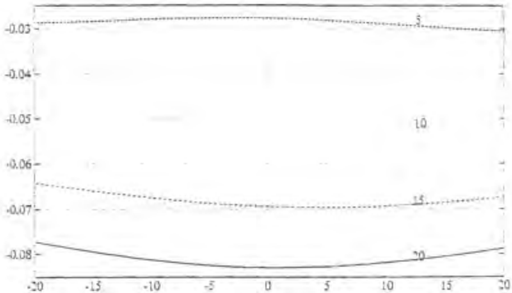


**Figure 52**: $\dot{\psi}$ vs Stern Plane Angle: $x_{cb}$=-.5%

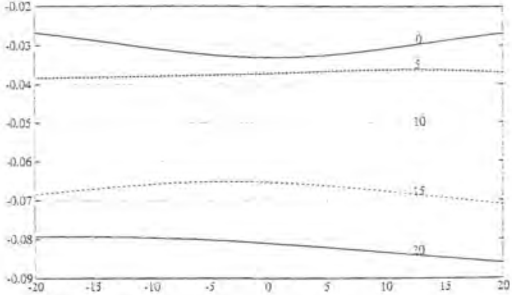


**Figure 53**: Stability Index vs Stern Plane Angle: $x_{cb}$=-.5%

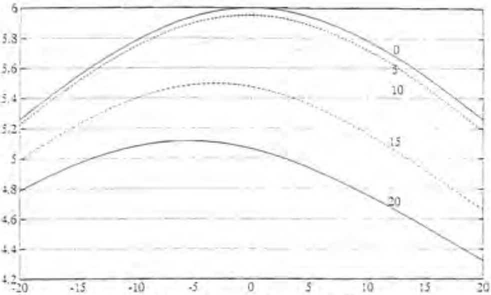

**Figure 54**: u' vs Stern Plane Angle: $x_{1s}$=-1.0%

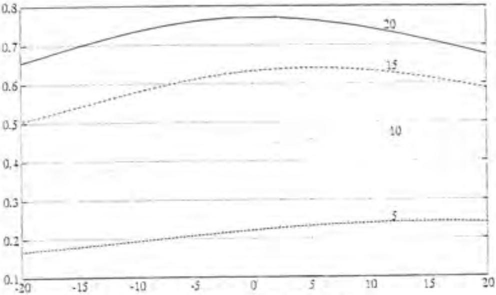

**Figure 55**: v vs Stern Plane Angle: $x_{1s}$=-1.0%

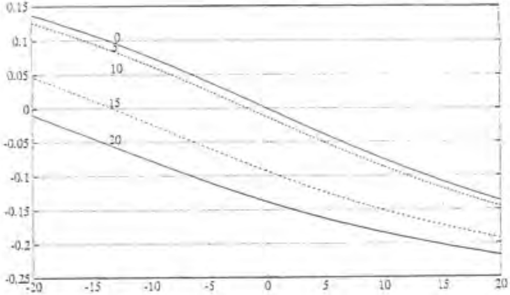


**Figure 56**: w vs Stern Plane Angle; $x_{cg}$=-1.0%

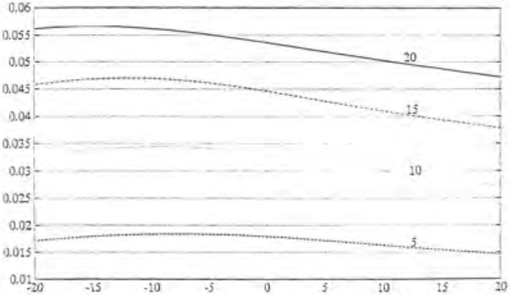


**Figure 57**: p vs Stern Plane Angle; $x_{cg}$=-1.0%

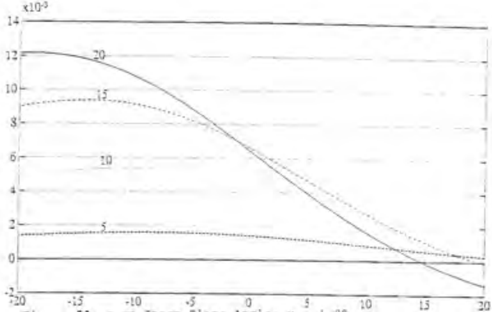

**Figure 58**: q vs Stern Plane Angle: $x_{cg}$=-1.0%

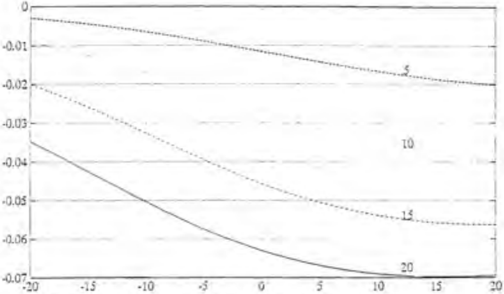

**Figure 59**: r vs Stern Plane Angle: $x_{cg}$=-1.0%

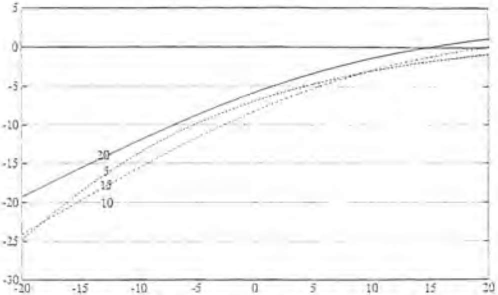


**Figure 60**: $\phi$ vs Stern Plane Angle: $x_{cg}$=-1.0%

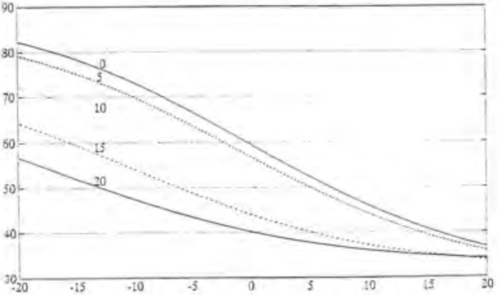


**Figure 61**: $\theta$ vs Stern Plane Angle: $x_{cg}$=-1.0%

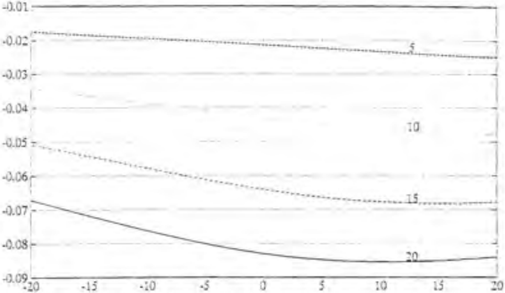


**Figure 62**: $\dot{\theta}$ vs Stern Plane Angle; $x_{cg}$=-1.0%

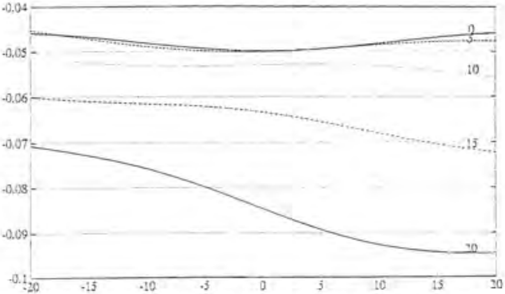


**Figure 63**: Stability Index vs Stern Plane Angle: $x_{cg}$=-1.0%

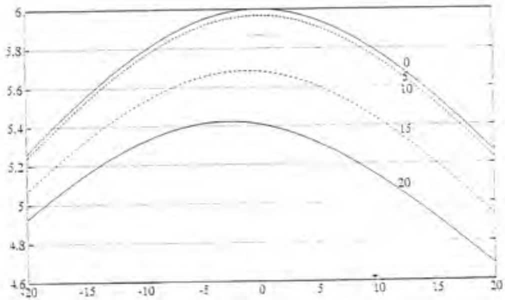


**Figure 64**: u vs Stern Plane Angle; $x_{cg}$=-1.5%

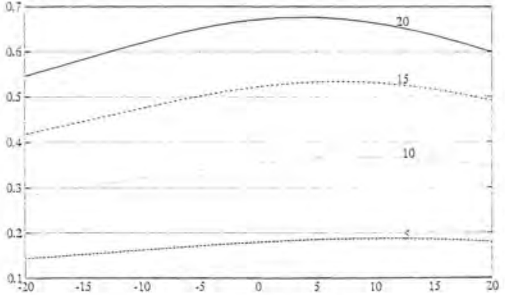


**Figure 65**: v vs Stern Plane Angle; $x_{cg}$=-1.5%

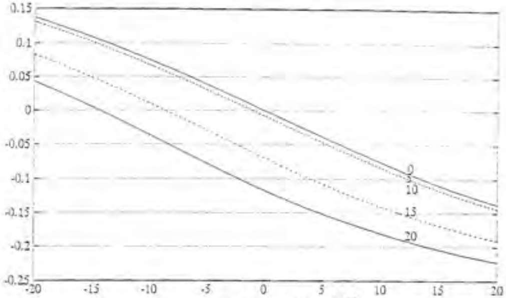


**Figure 66**: w vs Stern Plane Angle; $x_{cs}$=-1.5%

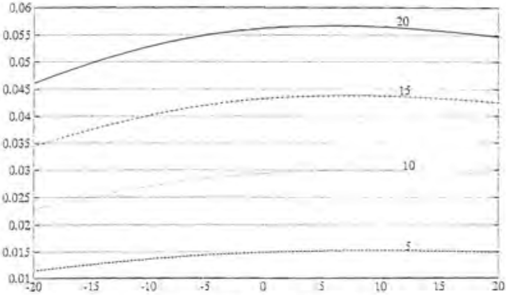


**Figure 67**: p vs Stern Plane Angle; $x_{cs}$=-1.5%

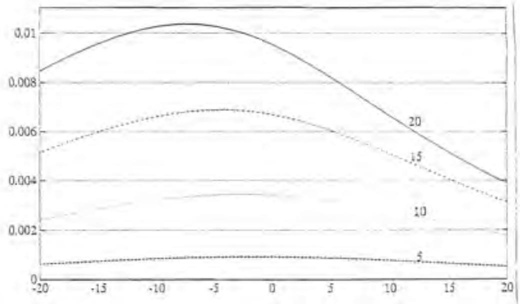


**Figure 68**: q vs Stern Plane Angle: $x_{GB}$=-1.5%

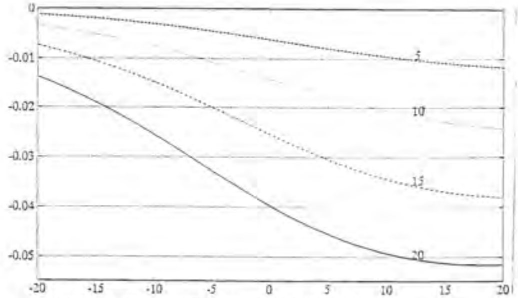


**Figure 69**: r vs Stern Plane Angle: $x_{GB}$=-1.5%

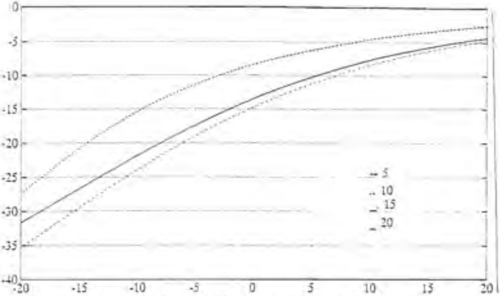


**Figure 70**: $\phi$ vs Stern Plane Angle; $x_{cg}$=-1.5%

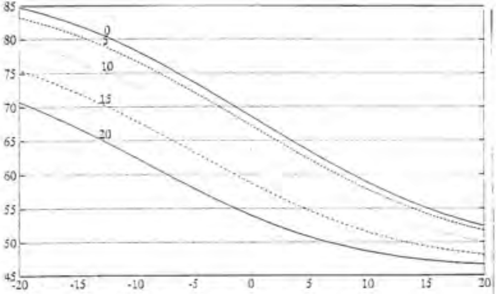


**Figure 71**: $\theta$ vs Stern Plane Angle; $x_{cg}$=-1.5%

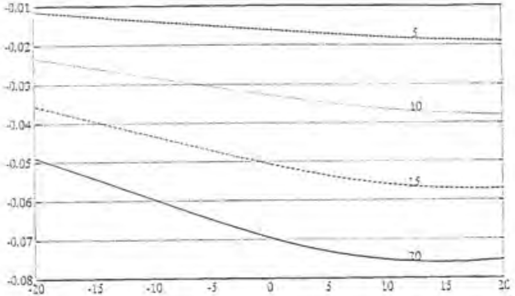


Figure 72: $\dot{\delta}$ vs Stern Plane Angle: $x_{GB}$=-1.5%

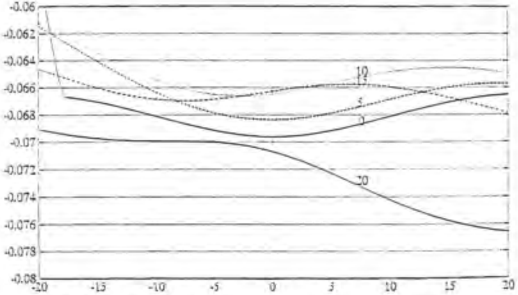


Figure 73: Stability Index vs Stern Plane Angle; $x_{GB}$=-1.5%

# V. CONCLUSIONS AND RECOMMEDATIONS

The results of the previous chapter show that the continuation algorithm is efficient in tracing the complicated steady state solutions associated with motions in six degrees of freedom. The translational and angular velocity components, the Euler angle components for roll and pitch and the and the rate of change of the vehicle's heading are all products of the algorithm. The degree of stability index was also presented in graphical form in order to determine if the vehicle would remain under positive control while maneuvering. The more negative the index, the more stable the vehicle motion. In every set of data the index became more negative as rudder angle was increased. Therefore, for each set, the straight line case was the least stable. This same effect, though not as pronounced, is present for non zero dive plane angles. Recommendations for future research in related areas include the evaluation of different control schemes with respect to performance. Some examples are the verification of vehicle path accuracy, the ability of the vehicle to reject external disturbances and remain on course, and the degree of confidence in performance and stability if there are errors between the physical and mathematical modeling of the vehicle.

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3. Papoulias, F.A.,"Dynamics of Marine Vehicles", Informal Lecture Notes for ME 4823, Naval Postgraduate School, Monterey, California, Summer 1993.

4. Smith, N.S., Crane, J.W.,and Summey, D.C., SDV Simulator Hydrodynamic Coefficients, Report NCSC-TM231-78, Naval Coastal Systems Center, 1978.

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